REPEAT

Abstract

The concept of repetition makes its appearance in our language, practice and education, both as a term of *belittling* and as a term of *praise*. We belittle those agents who repeat like parrots or monkeys, and we praise law-abiding citizens who repeat *(adhere to)* laws to the letter. I will argue that the readily available model we have for the following of rules is that of repetition and, further, I will argue that this model is at the core of Wittgenstein's analysis and critique of the picture of following rules that *"held us captive."* I suggest a conceptual framework wherein *mistakes* and *repetitions* are identified as one, and doing what is right (and what is true) is modeled on what I will term *extension*. I further argue that education is where we repeat, which is the equivalent of *exercising*, but that the point of the *educational exercise* is that we do not repeat in real life, but try and do what is right.

REPEAT

> A *picture* held us captive. And we could not get outside of it, for it lay in our language and language seemed to repeat it to us inexorably[1]

> I can imagine a Logician who tells us that he has now succeeded in *really* being able to *think* 2×2 =4. [2]

To follow a rule is to repeat. [3] This happens to be the common wisdom concerning the following of rules. An agent who follows rules repeats: that is, he keeps enacting the same thing throughout changing circumstances. A young student, who adds one to various numbers, repeats the same thing when he adds one to the number two or to the number three. In both cases (circumstances) the same thing is being followed, that is, repeated, however different the final result.

Let us start by suggesting the following maxim as capturing what Wittgenstein has told us about the subject of following rules: (1) doing the right thing is never a repetition. However, since a rule introduces a standard, thereby constituting what counts as right, it would seem that Wittgenstein is saying that to follow a rule, to repeat, is not to repeat. In this apparent contradiction, we have the core of Wittgenstein's paradox of following a rule. This short paper is devoted to the presentation of this paradox, by way of which I will also touch on the issue of Modernity.

From the opposite end, let us consider the case of a mistake. Here we are going to argue (2) that mistakes are repetitions. How is this possible? Could the student not come out on his own and for the first time with a mistake?

Suppose we ask the student to compute 2 plus 2. Suppose the student gets it right and answers 4. We could now pose this question to the student: why 4? In this context, the "why" question should not be confused with a "how" question, to which the student could answer by describing the way in which he conducted the computation. The "why" question, in context, inquires why this outcome, 4, and not another answer, say, 5? To this the student would have only one reaction: "Why 4? Simply, since this is the right answer."

Suppose that the student had, in fact, made a mistake. In answer to the question of the sum of 2 plus 2, he answers 5. What would then be the student's response to the "why" question? A moment of reflection must show that the student would have no easy answer, if any. The crux of the "why" question is: why 5, and not 7 or 3? Equally, all are mistakes. This is also true of a categorical mistake. For example, let us say the case when the student answers: 2 plus 2 is a family. Note that our argument does not depend on the correct answer being unique, with respect to which all other possible answers are mistakes. For example, the student might be asked to name an even number. If the student answers 2, and is then asked why 2, his answer is as before: because 2 is an even number. Before the correct answer, the "why" question makes little sense. The student have just done what he was asked: provide the right answer.

The point here is that, in putting the question to the student, a mistake was not intended for his reply. In that we are reminded of a simple truth, however obvious: that a

mistake is a failure. We do not plan for a mistake, but quite the opposite: we aim at success, and when we fail we call this a mistake. However, we have to admit that a student who answers wrongly does succeed in doing something. He seems to be producing a meaningful answer, however mistaken. It is here that the "why" question makes its impact. The student's mistake does make sense, i.e. there is—and in fact there must be—a previous question to which the student's mistake could have been given as the correct answer; otherwise, we would not have recognized the student's reply, whether a verbal utterance, an inscription on a piece of paper, a gesture or a sigh, as intended to deliver a correct answer to the question presented. [4]

We observe therefore that a mistake is a correct answer given to the "wrong" question—a valid answer, taken out of its proper context. The answer of "5," mistakenly introduced as an answer to the question with regard to the sum of 2 plus 2, is the right answer to a question regarding the sum of 2 plus 3. The student who answers "5" recites an old answer. He *repeats* an answer but, as it happens, he does this in the context of the "wrong" question. This could also be true of a categorical mistake. "A family" might be offered as the right answer to the question of what is the sum of two (parents) plus two (kids).

Hence, we say that all mistakes are of equal value before the "why" question. Certainly, within the context of a certain subject matter, some mistakes rather than others indicate more fully the student's grasp of the material being taught. In the teaching of mathematics, 5 as the sum of 2 plus 2 could indicate a better grasp of the subject matter than the numeral 7, or the answer "a family," can do. But what might be indicators for teachers to judge the standing of their student with respect to the material being taught,

might also present themselves as excuses when they are used by the student in order to explain his mistake. The student may offer to excuse his mistake, but none of the excuses is an answer to the "why" question, since what we are asking for is *a reason*. The student who answers correctly has the reason presented to him in the form of the question. In response to "the sum of 2 plus 2," he finds reason to answer: 4. However, the question as regards the sum of 2 plus 2 does not present a reason for any of the multitude of possible mistakes. *With respect to a reason, all mistakes are of equal value*. To conclude: whatever is offered as an explanation to a mistake is nothing but an excuse. And an excuse is not a reason. In fact, as we have argued before, we have ascertained that all excuses, offered as after-the-fact support of mistakes, can be explained away as rooted in repetition.

Consider a critically-minded agent who is confronted with a question on some political issue when he is unsure of the right answer. If we were to force him to come up with an answer, what might he say? If he comes up with an answer at all, he would be quoting, for example, the political column in the newspaper, the opinions of a friend or those of a local politician. In short, the agent will be repeating.

It would seem as if a student could also recite or repeat the right answer in the same manner as in a test, during which a student might copy the right answer from a fellow student. However, we may assume that a student who only utters the word "four" in response to "the sum of 2 plus 2" does not produce the right answer: he only repeats. If I were to answer a question in Latin by way of repeating what a Latin scholar whispers in my ear, I would not be producing the right answer, but only an imitation of the right answer. Indeed, the reader could insist that right answers exist in a manner independent

of context and origination. I might not be producing the right answers but, as it were, could still be giving or presenting the right answers. But what is the case when I am presented with a gold coin which I later realize is only painted gold: was I indeed given a gold coin? Also, in our case, the teachers might be under the false impression that a right answer was given, that the answer sounds and looks right, and only later discover that they were reading the right answer into, as it were, an empty frame. In this respect, reading the correct answer "4" in an utterance of "four" is not that different to reading it in an utterance of "five." This would hold true for an inscription on paper or for any other mode in which answers may be delivered.[5] Therefore, in order not to give the wrong impression about our intention in (1) and (2), we should also add (3): repetitions are mistakes.

Our conclusion is then an identification of mistakes with repetitions, resulting in the understanding that to do the right thing is not a repetition. But if repetitions and mistakes are one and the same thing, then what are non-repetitions, what we might call "extensions"? The extent to which Wittgenstein's later philosophy is radical becomes clear when we realize that the inverse of (1) also holds: that is, (4) any non-repetition or extension is the right thing. In other words, there is nothing more to correctness or being right than being non-repetitive: if you extend, you are right!

The careful reader might observe that both (1) and (4) were phrased in a general way: that of doing the right thing and not that of doing the right thing according to any specific rule. This is no source of real perplexity when we understand that if the act and the rule were two distinguished entities, then the act would be a repetition of the rule, and therefore a mistake. Therefore, to do the right thing is a constitutive act, a legislative act

in the Kantian sense. It legislates what is right by way of exemplification. Being legislation, it is not a repetition. If it is not a repetition, then there is no pre-existing question to which this could have been given as an answer. In other words, there is no pre-existing standard or legislation but the one associated with the question or context in question that this act exemplifies. This act exemplifies itself, and therefore must be right.

In answering the question about the sum of 2 plus 2, the student, if he answers correctly, exemplifies the rule of "plus." But in so doing, the student legislates the meaning of "Plus" before our eyes. If he does not, he is simply repeating. The "Plus" rule must be present in the *single* sample of its application (here this is 2 plus 2) in the same manner in which a color is present in the single sample of a colored object. If we think that nothing short of *all* possible applications of the rule are required in order to determine the meaning of the rule, we immediately expose ourselves to the possibility of non-standard interpretations of the rule, i.e. interpretations that coincide in values with the rule up to a certain point—say, a point which is out of reach of the capacities of agents—and thereafter differ in value. For example, in his analysis of Wittgenstein's following a rule argument, Kripke offers the concept of "Quus" as a non-standard interpretation of "Plus," and further forces us to agree that "no-condition in the world" could differentiate the agent's following of the rule of "Quus" rather than the rule of "Plus." [6]

However, if we assume, as we must, that the student who answers correctly does not repeat, i.e. he does not simply follow the (lexicographically) ordered list of tripled numbers expressing "plus," and lists in the appropriate place the ordered triple (2, 2, 4), then the difficulty is avoided.[7] In the question presented to the student, he finds a reason

and an opportunity to exemplify and, by the same token, to legislate what is meant by "plus." In exemplifying 2 plus 2, the student must apply a *fresh* reasoning that brings him from the initial step presented before him (the question), to the concluding step (the answer). This requires that the student recognizes that the question presents him with a difficulty or an obstacle (to reasoning) that must be overcome. If he does not recognize the question as presenting a difficulty, he will simply be repeating an answer, even if, as argued before, the answer sounds right, e.g. even if he answers "four." To *actually think* 2 plus 2 and arrive at the correct answer must mean that there is a base (which is, in general, associated with the particular context in which the question is asked) and, in addition, that the (logical) trajectory from the base to the right outcome is a particular way chosen in order to overcome the obstacle to reasoning. In exemplifying the question in the context that it is put before the student, the student must recognize that he actually justifies his answer, by what he does in order to overcome the obstacle to reasoning. In self-exemplifying the answer to the question (and, by the same token, self-exemplifying the question), he thereby secures a justification for his possession of the answer.

The student can arrive at *(think of)* the answer to the question in many ways. He may observe, for example, that there are *two* entities present in *two* concrete appearances of the letter "I," so that they become ordered one after the other thus: (I,I). He then orders one more such double appearance of the letter "I," by way of which he arrives at: [(I,I), (I,I)]. He recognizes that he is justified in disregarding the (inner) parentheses, and then he counts [I,I,I,I], from the start. He may envision a staircase, such that the initial floor is not numbered (i.e. it is his zero point of beginning), and he then climbs the stairs by missing every other stair. He then marks the stair at which he arrives after two

consecutive leaps, and counts the steps from the beginning. The point to these exercises is that there must be a self-exemplifying procedure, concrete or abstract, such that within its frame the question and the answer become one.

The context here is essential, since the context presents the difficulty that the student needs to overcome so that the answer is able to be manifested. In the context of some minimal algebra, the student may recognize that 2 plus 2 is a particular case of 2 times X, and would follow from here to multiply, let us say, the weight of 2 kilos by 2. If the context is indeed axiomatic, none of the previous self-exemplifying procedures would do. Here the question as regards the sum of 2 plus 2 is the equivalent of proving that the sum of 2 plus 2 is 4, and therefore the base is a set of axioms. The student recognizes the difficulty of how he can prove (or compute, for that matter) that the sum is 4 from a base consisting of: (i) a+0=a; and (ii) a+S(b)=S(a+b). In this case, the student would be required to observe the essential substitutions (manifestations of the axioms) required so that he can move from (i) SS0+SS0, along the logical ladder to (ii) S[SS0+S0] to (iii) S[S(SS0+0)] and, finally, to (iv) S[S(SS0)] and to (v) SSSS0.[8]

Once the obstacle to reasoning is overcome, since the question and the answer became one in the form of a self-exemplifying procedure, we could still envision that the answer is *repeated* and that the student enacts the self-exemplifying procedure once again. The difference in this case is that he now does this without the (constant) presence of the obstacle to reasoning: the procedure, as it were, is no longer executed *against* the obstacle to reasoning. The line of reasoning is now being *repeated mechanically.* The procedure is *frictionless* or *idle*. What keeps it in motion is the *(fading)* memory of a road or a procedure once taken to a successful end—that of coming up with the right answer.

But the question and the answer are no longer one, and they are no longer being *thought* or *exemplified*, but only *repeated*. The point here could be represented as follows: *to think is actually to rethink.* The moment a thought is exhausted as a means of overcoming an obstacle to reasoning, the thought could still be repeated mechanically, but, in this sense, *it is not being thought again*. A thought that is repeated is no longer a self-exemplifying procedure, and therefore, according to the criterion we laid out, it must be conceived as a mistake.

The alarmed reader might suspect that maxim (4), "any non-repetition or extension is the right thing," is taken out of some post-modern nightmare: whatever exemplifies itself is right! The argument could run as follows: whatever course of action one takes surely exemplifies itself, and thereby the principle laid out above in maxim (4) must render it as correct or right. However, this line of argument misses the central idea of this paper. For something to exemplify itself means that it and what it exemplifies are one and the same thing. We could put it as follows: there should be no gap between *what it is*, that it is that course of action, and *what it does,* that which it exemplifies. The case of repetition surely does not obey this constraint: what it is, is the answer given or the course of action taken in the relevant (true) context of application; what it does is to repeat a previous answer or, if one wishes, to imitate a standard previously laid out. In our case, the case of non-repetition, or that of self-exemplification, no such gap exists. When no gap exists, at that very moment we are presented with extension, which therefore must be right: we are presented with a moment of truth.

The fact that self-exemplification cannot fail or be wrong should be clear from the following trivial consideration: if it fails, it no longer self-exemplifies, e.g. when an

orator testifies to his momentary stuttering by clearly pronouncing: "I'm stuttering," he fails in exemplifying his condition. This is not true when the orator's utterance self-exemplifies itself, as in: "I'm st..utt..tttt..ering." It is worth noting that the idea of the proposition, the unit of content that is expressed by sentences of language, exhibits exactly this kind of structure. Wittgenstein is adamant in arguing that the proposition is a self-exemplifying procedure. The proposition offers itself as an identity-criterion for whatever could be the case: "A proposition shows how things stand if it is true. And it says that they do so stand." [9] Thus, when the proposition succeeds in exemplifying how things stand, it becomes true. By way of its becoming true, we are getting familiar with its content, with its sense, in other words with that which it exemplifies. The proposition, therefore, is a legislative act carried out by way of self-exemplification. What would then be the fate of a proposition that fails to exemplify, the case of a false proposition?

A moment of reflection will convince us that the false proposition is perplexing. If the false proposition is a failure in self-exemplifying, then how could it be said to fail, when *what it is* and *is said to fail*, has not yet been exemplified for it to fail?

Set in these terms, the false proposition presents no easy puzzle. However, our analysis of mistakes as repetitions offers us a way out. The false proposition is simply a repetition. What we repeat is what is left from the proposition when it fails. This would be the propositional sign, or, if one wishes, the very physical construct, the vehicle that was supposed to carry the self-exemplifying procedure that we call the proposition. The proposition was aimed at a fact by way of self-exemplifying that fact, or, in other words, by way of becoming this fact. Only then—when there is no gap between it (what it is) and its fact (what it does)—does the proposition self-exemplify its case (its fact, or its

reality). When it fails, the existence of a gap is recognized, and we are left with only the physical construct with which this move, aiming at factual truth, was to be carried out. However, this physical construct—one might think of it as a concrete sentence that expresses the proposition—also belongs to reality: "The propositional sign is a fact," Wittgenstein is quick to remind us.[10] The fact that we are left with when the proposition fails is not the fact the proposition was aimed to express: to this we have no access, since we failed in our attempt to encounter this very fact. However, in suggesting the propositional sign, which in and of itself is a fact, we have indicated a prior encounter with a fact, a successful execution of some other proposition. What we are doing in trying to project this success and failing, is only to repeat this previous success. The first time around, this was indeed a success (if not, we would not be in ownership of the propositional sign). The second time around (recognizing that we are back at the propositional sign) it is only a repetition of this previous success, and a repetition of a success, as the reader can guess, is by now a failure.

The point, then, is that whenever we establish a success, for example that today is a nice day, we are not repeating, we are extending. In other words, today becomes not a passive acknowledgment that it is indeed a nice day, but a new criterion, ready to serve us in the (logical) future for what would count as a nice day.

We saw that a *repeated success is a failure.* However, it is worth mentioning here the common machinery we use to transform failures into successes: the machinery of translation. Suppose we project the propositional sign, the sentence "Peggy Bob" onto reality, hoping to find that Peggy stands to the left of Bob in the police lineup. This is done, in the fashion that Wittgenstein suggests in his *Tractatus*,[11] by way of observing

that Peggy stands to Bob in the same manner that the two physical inscriptions, "Peggy" and "Bob," stand to each other on the printed line. To be able to do this, we use as our anchor a previous success: the already established success of the fact that the inscriptions of names, "Peggy" and "Bob," stand on the printed line in the way they do. If Peggy were to the left of Bob in the police lineup, the propositional sign projected onto reality would "stop nowhere short of the fact"—the fact that Peggy is indeed left of Bob in the police lineup.[12] This is no repetition: we moved from printed inscriptions on a printed line to suspects in a police lineup. In so doing, a new criterion for "standing to the left of" is offered: the way Peggy and Bob stand in the police lineup. We could use this new criterion, in fact this new candidate for a propositional sign, to check (backward, as it were) the way inscriptions on printed lines are set, or whether Peggy stands to the left of Bob in the political issues of the day.

However, the case of interest to us here is that of a failure: the case of a repeated success. When Peggy is not to the left of Bob in the lineup, we repeat a success. The first time around, we are in possession of the concrete fact that prescribes the manner in which the inscriptions of the two names stand on the printed line. This being a concrete and a tangible fact, we consider it a success. The second time around we find ourselves again in possession of it when it returns, as it were, empty-handed from its mission to the police lineup and back. This being the repetition of a success, we come to realize that it must indicate a failure. In other words, we cannot draw the conclusion that, in fact, Peggy stands to the left of Bob. No extension is offered here. We cannot, at least at this stage, use the actual circumstances of the police lineup as a criterion, a propositional sign or as

a means of description of any sort. Again, we could only repeat a previous success. Hence, confronted with a repetition, we acknowledge a defeat, a failure.

How can we translate a defeat into a victory? The logic of asymmetrical relations teaches us to translate an occurrence of failure (in this case, that it is not a fact that Peggy stands to the left of Bob) into a success: that Peggy stands to the right of Bob. The same would hold elsewhere. In some black-white version of reality, when a certain object does not come up white, we translate this failure into an acknowledgment that the object is black. The point about translation, however, is that it too should not fall into repetition. If translating "not-left" to "right" or "not-white" to "black" were repetitions, then these moves would be failures. It is only because in moving from "not-left" to "right" or from "not-white" to "black" we extend,[13] that these are celebrated as successes. The extension involved becomes clear when we realize that in the introduction of "left" or "white," no corresponding reference is made to "right" or "black." We introduce "left" by pointing to the way Peggy stands with respect to Bob, and similarly, we introduce "white" by way of pointing to the color of a certain white object. Translation, we can therefore sum up, is successful when it does not repeat. The under-determination of translation can be transformed here to this: translation cannot be modeled on repetition. For us, of course, this is only an indication of the possibility of translation. And this applies to any other common notion of translation. A good or correct translation of a work of literature is one which does not repeat, but extends—so much for the chimera of (strict) literal meaning.

The reader might pause here and wonder what use repetitions are at all, considering that they are mistakes? Why can we not just extend, and thereby do the right thing? Let us consider, say, young players holding a basketball for the first time, with no

prior knowledge or acquaintance with the game, having never watched the game played: could they start extending the game, that is, do the right thing on the court? It would seem not. The point is that our young players stand nowhere in a position for them to extend. The young players have not yet acquired a place from which they would be able to extend. So we realize that repetitions are what education is all about. The training stage for the young players of the game of basketball consists, therefore, of repetitions, or of the equivalent of repetitions, that is, exercises. Exercises or repetitions are failures or mistakes. Hence, we learn from mistakes. This might explain the reluctance of some teachers to grade papers or exercises with the best grade possible. A perfect repetition seems as elusive as it should be—which is not surprising, concerning the fact that a repetition is a mistake!

This also explains the modernist perplexity with the idea of perfection, at which much of Wittgenstein's critique of Modernity is aimed, for example his attack on logic as sublime or "perfect." [14] *A perfect repetition is an oxymoron.* An extension, surely, is not perfect, in the sense required, since it obeys no standard but its own. And a repetition as the failure that we now came to recognize that it is, can hardly be rendered perfect. This, as a side remark, is the impossible predicament in which Modernity found itself in its twilight times: attempting to figure out a stand between the initial quest for a perfect repetition—what I once labeled, *"Perfecto-Mobile,"* the modern parallel to the phantom idea of *"Perpertu-Mobile"* of the Middle Ages[15]—and the postmodern acknowledgement of the failure of this project. Since, if doing the right thing is modeled on repetition, then any repetition, any failure, can be reintroduced as the right thing. Defeats, that is, what history—as written by the winners—spells out as failures, are thus reintroduced as

victories or successes when written by the losers. Simply, a change in perspective is a change in a standard. We change the standard to accommodate the failure, and thereby turn it into a success.

What, then, makes an exercise a repetition, and why is it doomed to be a mistake? The point to recall is that the actual circumstances of an exercise are not the circumstances for which the exercise is being enacted. Soldiers in the training stage practice with cartoon targets that only represent the possible enemies they are preparing to confront in the battlefield. So do basketball players who practice with their teammates in preparation for a league game. The presence of this gap is what makes for repetition. Indeed, we could say that indeed we repeat the same exercise over and over again, only because we can manipulate the gap as it suits us. We say that these actual circumstances represent the future circumstances for which we wish to be prepared, so we decide what we will count out: the steepness of the hill, the temper of the player we practice against, the specific number to which we add one. Much in the same manner in which we claim that all these are models of this car, and in so doing, we disregard their different sizes and different colors. It is thus that repetition is ensured as a manner of decision:[16] we decide that these different exercises are exercises of the same thing, that they are repetitions of the same thing. This is not different from deciding for a young student that he or she must enroll in a range of different educational activities, from athletics to a piano class or a drawing lesson. We decide for the young student that all these repetitions or exercises are his or hers, and we expect that a new category, a category that belongs to the future, would emerge. We hope to see in the young student's future the birth of a new category,

in which the all seemingly different exercises of his youth are conjoined as essential constituents. [17]

It is thus that, in the context of an exercise, we repeat or imitate certain circumstances. We act as if these were our circumstances, while clearly they are not. We could phrase this as follows: in the setting of an exercise, there is a *gap* between *us* and *our circumstances*. This is to say that in such a setting there are two "we"s in play: firstly, there is the "we" of us as trainees, the "we" that is exercising. As trainees we operate as we expect we should act in the capacity of full participants in reality. Thus, the "we" that we are as trainees must be one with the simulated circumstances, and therefore this "we" must be living in a gap: the gap that lies between it and its actual circumstances. The second "we" is the "we" that observes and expects that we simulate successfully—and thus, from the other end, this "we" also lives in a gap, a safe distance away from the circumstances that are simulated.

We also should mention that this is the gap inherent to abstraction. In exercising we abstract: we climb *a* hill, we play against *a* player, and we add one to *a* number. To do so, we abstract the "we" that observes from the "we" that acts. This gap between us and our circumstances, the *gap* of *abstraction,* is exactly the gap we must overcome when moving from the status of a student or a trainee to that of a full participant. In this move, we move from repetition to extension: from being a student of a rule, to the master of the rule. And in mastering the rule, we do not repeat, we extend.

So, extension lies in diminishing the gap between us and our circumstances, our reality. Of course, we have to be well prepared to make this move. When we are not, we

fall into repetitions, we live a life of a gap, when there should be "no-gap": this is the hatching place for (real-life) mistakes and failures.

What can we say about the living a life of a gap where there should be a gap, in other words, living the life of a student when one should and can? After all, the reality of the training or the educational phase must be as concrete as anything else in life. The sense in which we as students are indeed full participants of this reality, is the sense in which we observe ourselves as trainees: the sense in which we observe ourselves as agents who repeat. This explains the magic spell the picture of repetitions casts on us: we actually observe ourselves repeat. What more is needed as a convincing testimony that the human condition is organized around repetition? Indeed, these observations, the observations of repetitions makeup our education. As students, we learn from observing repetitions: we learn from our repetitions. Thus we do not and should not learn to repeat. As students we repeat, and in living the circumstances of a student fully, we observe what we do, that we repeat. But in observing repetition, repetition stops, and in this way we learn from repetitions, that is, we extend: we move on. Growing up, we should say, is therefore all about giving up on repetitions. Again, this is possible only if we (as trainees) repeat: thus we repeat so as not to repeat—something which indeed makes learning and the practice of following rules the paradoxical phenomenon that it is.

Notes

1. Wittgenstein, L. (1953) *Philosophical Investigations*, ∫ 115. Henceforth PI.

2. Wittgenstein, L. (1977) *Remarks on Colour*, ∫ 69, ∫ 109.

3. I was inspired to write this paper after watching a war documentary, in which I heard the following quote of an American Air Force Colonel, reflecting on his days as an Ace combat pilot in the Korean War: "If you did not want to shoot down MiGs in Korea, the best you could do is to follow the rules."

4. Not to say that a gesture or a sigh cannot be rendered meaningful—only that to consider them meaningful requires they could be construed as an answer to a question.

5. The dream of having a privileged notation or way of presenting answers such that no misreading is possible is therefore strongly discarded here.

6. The paradox is generated by introducing the concept "Quus" that coincides in values with "Plus" up to a certain (big enough) number, and thereafter it gives non-standard values, e.g. for x and y greater than the number in question, it gives a constant number, say, 5 as a value. In a carefully laid our argument, Kripke demonstrates that all possible candidates, e.g. dispositions, agents' intentions, divine observers looking into the agents' mind and more, would not provide for a condition in the world that could differentiate the agents' following the rule of "Plus" rather than that of "Quus."

See in: Kripke, S. A. (1982) *Wittgenstein on Rules and Private Language*.

7. The resolution offered here to Kripke-Wittgenstein's paradox is not that of the agent having "private sensations" or "private definitions" of the rule of "Plus," but rather that the agent *exemplifies* the rule. The import of "exemplifying" in this context has transcendental twist to it, which we should consider at some length. Let us reflect for a moment on the structure of Kant's transcendental argument. The argument has the following form: (i) E: Experience is such and such; (ii) C: certain Conditions are *necessary* for the possibility of experience as such, i.e. E→C; (iii) we have, E, E→C, and therefore we conclude C. Note that the argument has an empirical base as its premise (E). Hence, if we change our experience, that is, science, e.g. if we reject Newtonian physics in favor of Relativity theory, we must modify the transcendental conditions (C). We could (transcendentally) derive a different conception of space than the one held by Kant, without falsifying the form of his argument, e.g. if our premise (E) is the scientific experience entailed by Relativity theory.

Now the "Plus-Quus" paradox has a similar structure. Consider: (i) P: one knows, understands or grasps what "+" stands for; (ii) Q: certain conditions are *necessary* for the possibility of the ascription of the grasp of the arithmetical operation of "+" to an agent, i.e. P→Q. The condition Q is the fact that we can "*Quus"* the agent's beliefs, i.e. the fact that employments of "+" could be reinterpreted as employments of "⊕"—"⊕" is the symbol representing "Quus"—without, as it were, leaving a trace. In this new understanding, the paradox is not an argument which undermines the concept of meaning, but instead an argument which (transcendentally) re-enforces the possibility of meaning. It only specifies the transcendental conditions under which such attribution of the meaning of something (the operation of "Plus") by something ("+") to an agent is possible. A similar reasoning is at work in Kant's original argument. Could any piece of experience refute the condition C specified in Kant's argument? The answer is in a sense "no." Since if we are in possession of experience that refutes C, we must either reject this as coherent experience or modify the conditions C. Simply, the transcendental argument presents a criterion for what would count as experience. Similarly, if there was a "condition in the world" that could distinguish the ascription of Plus-belief from the ascription of Quus-belief to the agent, then we would either have to reject the assumption that the agent is in grasp of "Plus" (that he knows what "Plus" means) or we would have to modify Q, i.e. conceive of another concept which presents itself as a (transcendental) pre-condition

for the ascription of meaning "Plus" to the agent. There must be such Q-concept; otherwise, assumption P must be dropped.

Consider, for example, the ascription of the concept "x is black." The argument presented above necessitates that there must be a Q-concept, such that all ascriptions of black-beliefs to the agent could be reinterpreted (indiscernibly) as ascriptions of Q-concept beliefs. In a simplified black-white version of the universe, such a Q-concept could simply be the concept expressed by "x is not white." It would be absurd in this context to expect that there is a condition in the world that would distinguish between black-beliefs and not-white-beliefs. Since if there is such a condition, then there must be a case where the agent judges that something is black, yet it is not not-white, i.e. a belief of a certain object that it is both (all over) black and white—which is impossibility. This implies that if the agent insists that he encounters an object which is both black and white, we would withdraw with the assumption that the agent in question masters the concept "x is black" (or "x is white," for this matter).

The misconception of the real import of the "Plus-Quus" paradox is due to the fact that it is conceived as having a form which is analogous to that of "an indirect proof" ("proof by contradiction"). We assume (i) P: the agent understands what "+" stands for; and we interpret the possibility of "Quus" as introducing a paradox; none of the four possible moves: ascribing the agent with either of the two beliefs (the Plus-belief or the Quus belief, but not the other), both or none, would seem to make sense. (In fact, formally, we have, P, $P \rightarrow Q$, Q, and hence, $Q \wedge P$, which we consider as a contradiction.) We are forced, therefore, to withdraw with "realism" with respect to the concept of meaning, i.e. with assumption (i), since it is shown to be generating a contradiction (our paradox). But Wittgenstein's argument is a transcendental argument and not "proof by contradiction." Premise (ii) does not falsify (i), but rather presents a transcendental condition for the very application of (i).

Now how does the argument of the above connect to the idea of exemplification? The student who *exemplifies* or *thinks* "Plus," say, in the context of 2 plus 2, could be reintroduced to the idea that he is, in fact, "Quus-ing." The student would therefore need to *rethink* 2 plus 2 in this new way. If he cannot, it must mean that we were simply wrong to assume that he actually was *thinking* or *exemplifying* "Plus" in the first place.

8. (a)=a

9. Wittgenstein, L. (1921) *Tractatus Logico-Philosophicus*, ∫ 4.022. Henceforth TLP.

10. TLP ∫ 3.14.

11. For a more elaborate account of propositions and facts in the *Tractatus*, the reader is referred to my, …

12. This key Wittgenstenian phrase is taken from his *Philosophical Investigations,* ∫ 94. The original sentence reads: "When we say, and *mean,* that such-and-such is the case, we—and our meaning—do not stop anywhere short of the fact."

13. In more traditional terminology, this would be translated into taking these moves or "identities" to be in the Kantian sense synthetic and not analytic.

14. See PI: ∫ 81: "As if our logic were, so to speak, a logic for a vacuum"; ∫ 89: "In what sense is logic something sublime?"; ∫ 97: "Its essence, logic, presents an order…—it must rather be of the purest crystal"

15. See my own paper, …

16. I use "decide" here for the lack of a better word. Understanding of this notion will hopefully become clearer in time. Here I will only mention that "decisions" are reserved solely to education. In this sense, education is an experiment. But life demands extension: for the individual who extends, there were no decisions to make. To put this somewhat paradoxically: the agent, who extends, does not move—for he or she has no other option but the one that is his or her own—what we later in the paper call: being one with the (or with one's) circumstances. This is the context in which the words of Franklin Delano Roosevelt resonate so well: "There are many ways to move forward, but only one way to stand still."

17. The new category would contain all the different repetitions/exercises culminating at the moment of extension: there we will have a birth of a new category in which all the seemingly different exercises join together as essential constituents—not, as it were, as ingredients in a mixture, but as indispensable contributors to the birth of the genuinely fresh category. The new category enables us to see as one, what before the moment of extension were distinguished entities: we will now see them as manifestations of one and the same thing, jointly classified under the same heading. This is the exact sense in which we should recognize that classifications and re-classification are byproduct of extension. In other words, extension is the way in which old classifications give rise to new ones. Also for the young student, all the different lessons s/he was taught during her/his youth form a new category: what the student as a grownup is or become—if only s/he extended.

References

Kripke, S. A. (1982) *Wittgenstein on Rules and Private Language*, Cambridge: Harvard University Press.

Wittgenstein, L. (1921) *Tractatus Logico-Philosophicus*, Ogden C. K. (Tr.), Harcourt, Brace and Company, London: Kegan Paul, Trench, Trubner & Co. Ltd. (1933).

Wittgenstein, L. (1953) *Philosophical Investigations*, Anscombe, G. E. M. (Tr.), Oxford: Basil Blackwell.

Wittgenstein, L. (1977) *Remarks on Colour*, Anscombe, G.E.M. (Ed.), McAlister, L. and Schattle M. (Tr.), University of California Press, Berkeley and Los Angeles.